A Study of Market Share in Duopoly Market Structure

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Abstract

Duopoly market structure often occurs in Hong Kong across different industries. Market share would be the most essential factor affecting the chance of survival in the market. In this paper, we study what parameters are more influential to the market share. A numerical model in terms of quantity demand and price will be introduced for our study. Then we apply the model with assumptions to study the numerical effect of parameters on market share Finally, we raise some key phenomena of the duopoly market share and suggestions for the company under different scenarios.

1 Introduction & Objectives

In Hong Kong, we can easily notice there are many duopoly market structures in different industries surrounding us such as:
- **Airline Industry**: Cathay Pacific Airways Limited and Hong Kong Airline Limited
- **Telecommunications**: PCCW (HKT) and Hong Kong Broadband Network (HKBN)
- **Supermarket**: Wellcome and ParknShop

In this project, we have researched two industries to study what are the major stakeholder market share and what elements are observed in a duopoly market structure.
Bus Industry:
There are 3 major parties in the bus industry: Kowloon Motor Bus Company(KMB), Citybus Limited and New Lantao Bus Limited.
The market share of the bus industry is calculated in terms of number of bus routes. The total number of bus routes in 2024 is 699. KMB has occupied 448 which is 64%. Citybus Limited occupied 203 which is 29%. Lastly, New Lantao Bus Limited occupied 29 which is only 4%. Therefore, the bus industry consisted of only two major parties which are KMB and Citybus Limited. They occupied over 96% of the total market.share.
Airline Inudstry:
There are 2 major stakeholder in Hong Kong Airline Industry: Cathay Pacific Airways Limited and Hong Kong Airline Limited. The rest market share are occupied by different regional carries.
The market share are calculated in term of number of seat capacity occupied by the airline company in Hong Kong International Airport. Cathay Pacific Airways Limited occupied 60% of the seat capacity while Hong Kong Airline Limited occupied 25% of the seat capacity.And the rest market share are occupied by different regional carries.

So this is crucial to study the effect of different factors that influence the market share. Therefore, We would like to find out what parameters would be more influential on the market share.

2 Background of Research

2.1 Assumptions

After conducting research. We decided to apply the model made by O. DIAZ–RODRIGUEZ, & T. NGUYEN, & H. KIM,& T. SENDOVA[1]. The model is derived by ODE and based on the following assumptions:

1. Two companies completely dominate the market.
2. Two companies sell homogeneous goods.
3. Company A will adopt a more aggressive pricing strategy than its rival.
4. The quantity demand and price are directly proportional to the quantity of the market share.

2.2 Simple Model Include Quantity Demand

From the above assumption, suppose there are two competing companies A and B in the market. Then we have:

$$A(t) + B(t) = 1 \tag{1}$$

Thus

$$B(t) = 1 - A(t) \tag{2}$$

Where A(t) and B(t) are the market share of the company A and B at time t respectively. Then we can construct the simplest market share model including quantity demand.

$$A'(t) = -\beta A(t) + \alpha B(t) \tag{3}$$

$$B'(t) = -\delta B(t) + \gamma A(t) \tag{4}$$

A'(t) & B'(t) are the rate of change in market share of the company A and B at time t respectively. α and δ are the quantity demand of company A and β and γ are the quantity demand of company B. The first term in equations (3) and (4) represents the impact of the rival's quantity demand on the company's market share. The second term represents how much the company's market share will increase by attracting the rival's customers.

2.3 Quantity Demand of a Gaussian Distribution

To facilitate model construction, we make an additional assumption that the relationship between price and quantity demand follows the standard normal distribution as Figure 1[2].

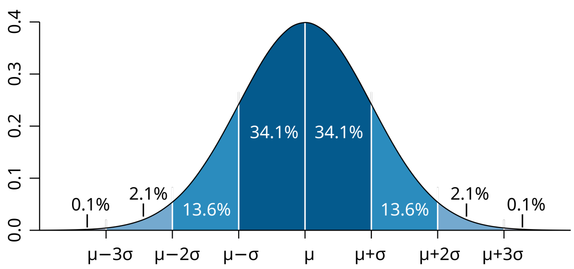


Figure 1: Standard Normal Distribution curve

Then the quantity demand of the company A & B is

$$\alpha = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(P_A-m)^2}{2\sigma^2}}, \quad \beta = \frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(P_B-m)^2}{2\sigma^2}} \tag{5}$$

σ is the standard deviation. m represents the average price of the goods. P_A and P_B represent the prices in terms of market share for the company A & B respectively. substituting (2),(5) into (3), we have

$$A'(t) = (\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(P_A-m)^2}{2\sigma^2}})(1 - A(t)) - (\frac{1}{\sqrt{2\pi\sigma^2}} e^{\frac{-(P_B-m)^2}{2\sigma^2}})(A(t)) \tag{6}$$

Where

$$P_A = kA(t) + m - 2\sigma, P_B = jB(t) + m - 2\sigma \tag{7}$$

k & j represent the increasing rate of change in the price for the company A & B respectively. Under standard normal distribution, two standard deviation cover 95.4% of the cases between price and quantity demand. So for P_A & P_B, we set a price floor equal to m - 2σ to prevent both companies set up their price of goods lower than m - 2σ.

Since our goal is to study the relationship between pricing strategy and market share for one company. Then we will introduce a new ratio r = k/j. Then we have.

$$P_A = rjA(t) + m - 2\sigma, P_B = j - jA(t) + m - 2\sigma \tag{8}$$

To facilitate calculation, we simplify the exponential function (6) by using Taylor expansion. After substituting other variables into (6), we get a desired equation describing two competing dynamics with the quantity demand of Gaussian distribution:

$$A'(t) = (\frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(rjA(t) - 2\sigma)^2}{2\sigma^3\sqrt{2\pi}})(1 - A'(t)) - (\frac{1}{\sqrt{2\pi\sigma^2}} - \frac{(j - 2\sigma - jA(t))^2}{2\sigma^3\sqrt{2\pi}})(A(t) \tag{9}$$

In this model, we will focus on the impact of two parameters j & r on the market share. j is the rate of change in price for company B. r is the ratio of the rate of change in price for company A divided by the rate of change in price for company B. For example, company B increases the price at a rate of $3 per week and company A increases the price at a rate $6 per week, the ratio R would be 6/3 = 2. This ratio reflects the similarity of the two pricing strategies. When R is closer to 1, it means the two companies' pricing strategies are more similar. In the following study, we will input different j & r values to observe which parameter will have a larger effect on market share.

3 Numerical Results

We generate the numerical result of this model through MATLAB.

3.1 Case 1: r = 1

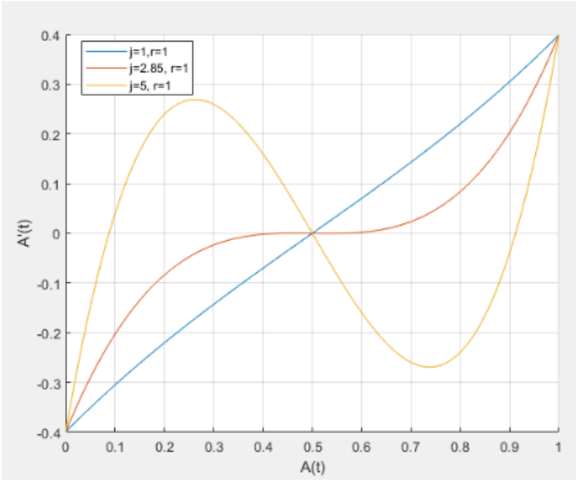


Figure 2: Rate of change in market share for company A When r = 1.

Figure 2 shows the rate of change in market share when r = 1. r = 1 means company A will fully follow the pricing strategies of company B. If the model has one equilibrium point, then when A(t) is less than the equilibrium point. A'(t) will always be negative thus A(t) will converge to 0. When A(t) is larger than the equilibrium point, A'(t) will always be positive thus A(t) will converge to 1. Therefore, both companies could stay in the market only if their initial market share equals the equilibrium point. Otherwise, the result of company A is to achieve a monopoly or drop out from the market. This means the market is volatile under this pricing strategy.

Also, the equilibrium point serves as a threshold market share level. For instance, when j = 1, if the initial market share of company A is larger than 0.5, then company A can eliminate its rival by increasing the price at rate = j × r = 2 per time t. If company A wants to survive in the market, it must obtain a certain level of initial market share, which is the single equilibrium point. Therefore, the chance of company A to survive in the market is inversely proportional to the equilibrium point.

This result is similar when the model has two distinct equilibrium points as A'(t) is still positive when A(t) is larger than the two equilibrium points. The only difference is both companies could survive if their initial market share equals one of the two equilibrium points.

Now if the model has three distinct equilibrium points, denoted as A_1, A_2 and A_3 in ascending order. When A(t) is smaller than A_1 or larger than A_3, A(t) will converge to 0 and 1. But generally, A_1 is very small and A_3 is very large. For instance when j = 5, A_1 and A_3 are 0.08 and 0.91. This means that company A is strongly dominating or being dominated in the market. This is close to a natural monopoly. It's trivial to analyze this because our target is to analyze how the rate of change in price affects the market share.

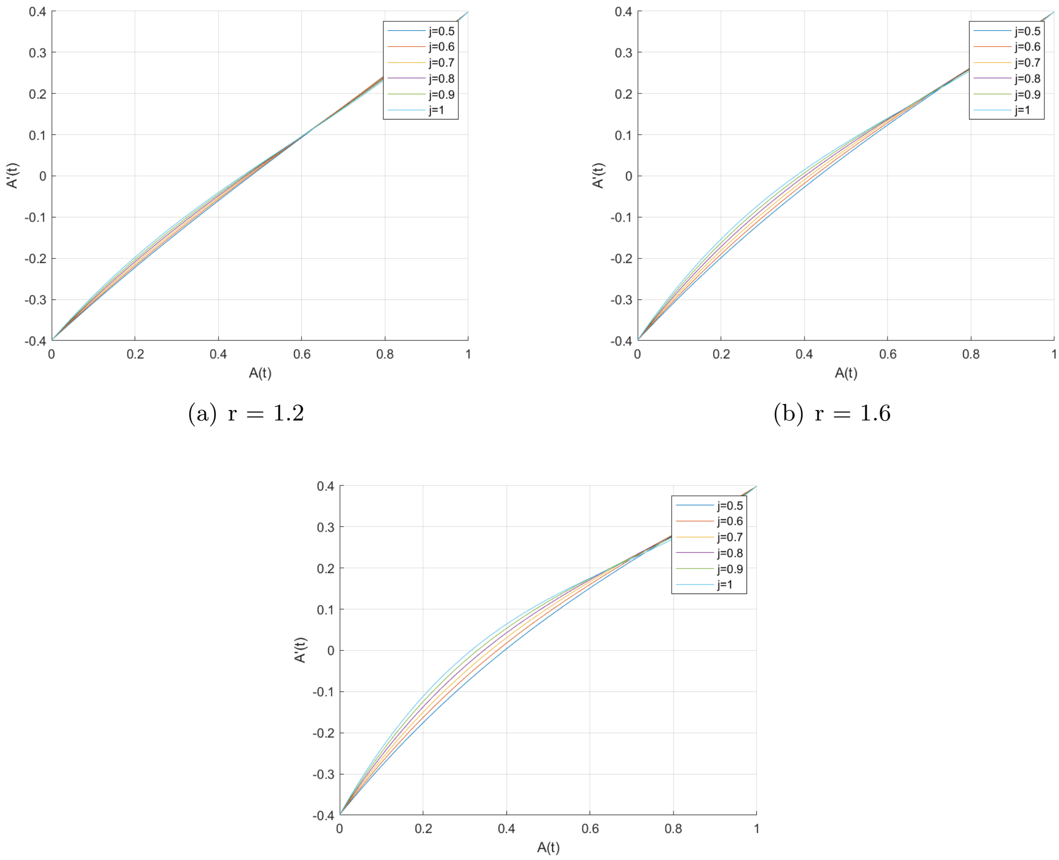
When A(t) ∈ [A_1,A_2], A'(t) will always be positive. But there exists a local maximum in (A_1,A_2). This means A(t) will not strictly increase to 1. After A(t) passes through the local maximum, A'(t) will decrease until it reaches 0, which is the second equilibrium point. Similarly, when A(t) ∈ [A_2,A_3], there exists a local minimum. After A(t) passes through the local minimum, A'(t) will increase until it reaches 0. Combining two conditions, when A(t) *in*

[A_1,A_3], it will eventually converge to A_2. So we can conclude that in most cases, the highest possible market share for company A would be the second equilibrium point. Since the majority of A(t) can converge to this equilibrium point. This means both companies can survive in the market with a wide range of initial market share. Therefore the market is stable under this pricing strategy.

Besides, from Figure 2, we can observe no matter how j changes, the equilibrium point of the model stays at 0.5. This is the major reason we assume r>1 as it cannot reflect the relationship between the rate of change in price and market share.

3.2 Case 2: r>1, j low

In the following case, we consider r is strictly larger than 1. To figure out the effect of different pricing strategies on market share. We will implement three different values of r, which are 1.2, 1.6 and 2.0 to represent different pricing strategies of company A. r = 1.2 means the pricing strategy is the most conservative and r = 2.0 means the pricing strategy is the most aggressive. Now moving on to the choice of j. In reality, the rate of change in the price of bus companies is low. In 2024, the proposed Hong Kong bus fare increase rate ranges from 6.5% to 9%[3], which is around \$0.5 to \$1 Hong Kong dollar. We tested all possible rates of change in price in this range. The following table and figure illustrate the rate of change in market share of company A for each value j under different pricing strategies.



(a) r = 1.2 (b) r = 1.6

(c) r = 2

Figure 3: rate of change in market share when j is low

Pricing Strategies	j = 0.5	j = 0.6	j = 0.7	j = 0.8	j = 0.9	j = 1
r = 1.2	0.477629	0.473573	0.469577	0.465607	0.461625	0.45759
r = 1.6	0.434688	0.423254	0.412108	0.401144	0.390264	0.379365
r = 2	0.395074	0.377768	0.361266	0.34542	0.330108	0.315232

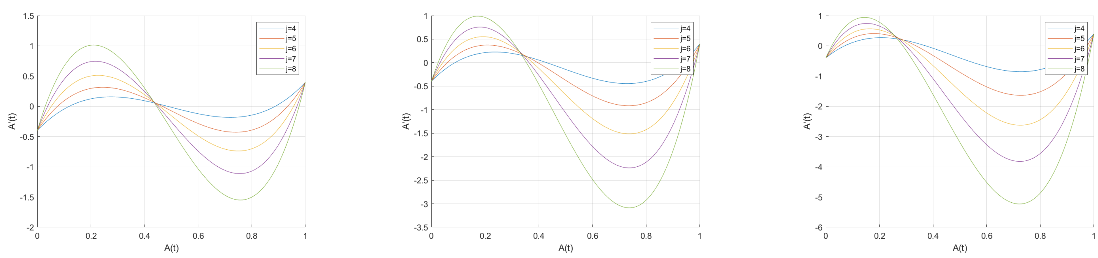
Table 1: The equilibrium point under different pricing strategies when j is low

From Figure 3, we can observe when j is low, no matter how company A choose its pricing strategy. The model has only one equilibrium point. So the market is volatile if the rival's rate of change in price is low. Take (r,j) = (2,0.6) as an example. Suppose the rival increases the price at a rate of $0.6 per year, if the initial market share of company A higher than 0.377768, which is the equilibrium point. Company A could eliminate its rival by increasing a higher rate of $1.2 per year. Therefore company A should pursue a lower equilibrium point for a higher chance of survival in the market.

Holding each j constant, when r increases from 1.2 to 2, the equilibrium point will decline by 0.082555 to 0.142358 depending on the size of j. Meanwhile, holding r constant, when j increases from 0.5 to 1, the equilibrium point will decline by 0.020039 to 0.079842 depending on the size of r. These data indicate that when j is low, the change in r has a slightly higher impact on the equilibrium point compared with the change in j. Also, the equilibrium point is inversely proportional to both j and r.

3.3 Case 3: r>1, j medium

In this case, we increase the value of j under the same pricing strategies for company A to observe the rate of change in market share. The result is shown below:



(a) r = 1.2 (b) r = 1.6 (c) r = 2

Figure 4: rate of change in market share when j is medium

Pricing Strategies	j = 4	j = 5	j = 6	j = 7	j = 8
r = 1.2	0.486104	0.466614	0.456496	0.449902	0.445129
r = 1.6	0.42912	0.396237	0.376898	0.363702	0.353956
r = 2	0.372799	0.336581	0.314167	0.298584	0.286988

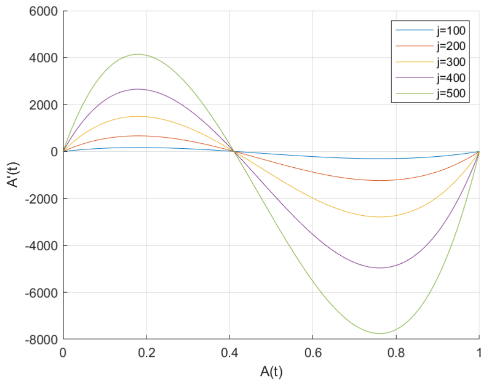
Table 2: The second equilibrium point under different pricing strategies when j is medium

From Figure 4, we observe when j increases to a certain level, the model will have three equilibrium points. It means the market becomes stable thus the highest possible market share of company A would be the second equilibrium point. In this case, company A should pursue a higher second equilibrium point to maximize its market share.

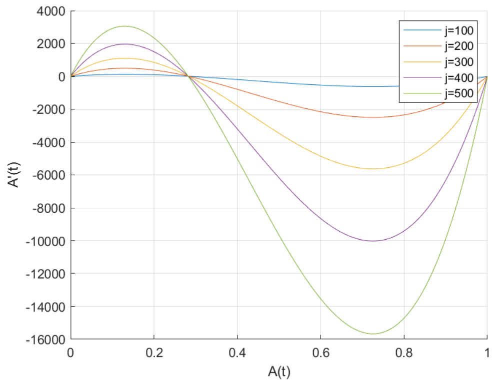
Holding each j constant, when r increases from 1.2 to 2, the second equilibrium point will decline by 0.113305 to 0.158141 depending on the size of j. Meanwhile, holding each r constant, when j increases from 0.5 to 1, the equilibrium point will decline by 0.020039 to 0.079842 depending on the size of r. Both impacts of j and r on the equilibrium point are slightly higher than in the previous case. But the relationship between the equilibrium point and j&r remains the same.

3.4 Case 4: r>1, j high

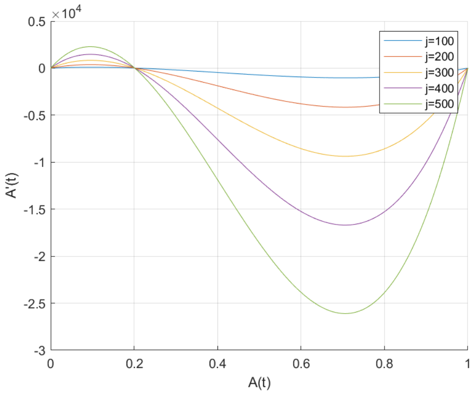
The airline industry is always considered a duopoly market structure in Hong Kong. And the price increase rate ranges from 8% to 10% which is around$100 to $500 Hong Kong dollars. We decided to set j equals to five different hundred dollars. The result is shown below:



(a) r = 1.2



(b) r = 1.6



(c) r = 2

Figure 5: rate of change in market share when j is high

Pricing Strategies	j = 100	j = 200	j = 300	j = 400	j = 500
r = 1.2	0.413056	0.41146	0.410922	0.410652	0.410489
r = 1.6	0.287524	0.28424	0.283133	0.282577	0.282242
r = 2	0.207858	0.203964	0.20265	0.201991	0.201594

Table 3: The second equilibrium point under different pricing strategies when j is high

Even though Figure 5 looks similar to Figure 4. The impact of j and r on market share has a large variance. Holding each r constant, when j increases from 100 to 500, the second equilibrium point will only decline by 0.002567 to 0.006264 depending on the size of r. This means the highest possible market share of company A almost remain unchanged no matter how j increases. On the contrary, holding each j constant, when r increases from 1.2 to 2, the second equilibrium point will decline by 0.208272 to 0.208895 depending on the size of j. Which is a significant change.

This result shows that when j is high, the change in r has a much higher impact on the equilibrium point compared with the change in j. Besides, when the similarity of the two companies' pricing strategies remains unchanged. Consumers are likely to stick with their original choices whether the rival's rate of change in price is high or low.

4 Limitation

There are two limitations to this model. Firstly, it ignores the impact of other substitutes. For instance, when the bus fare for company A increases. Apart from taking the rival bus, Passengers can take MTR as well. Therefore, increasing prices will lead to additional loss of consumers. This will affect the market share of both companies. Secondly, this model didn't show the direct relationship between price and market share. We can't observe under what price level company A will obtain the highest market share through this model.

5 Conclusion

By this model, we find some key duopoly market phenomena. First, the impact of the rival's rate of change in price on market share is weaker than the similarity of pricing strategies between the two companies. Then Under similar pricing strategies, consumers are more willing to stick with the original consumption habit instead of finding a new substitute. Furthermore, when the market is unstable, the company should adopt an aggressive pricing strategy. Meanwhile, when the market is stable, the company should adopt the most similar pricing strategy to the rival's strategy so that the company can maximize its highest possible market share.

References

[1] O. DIAZ–RODRIGUEZ, & T. NGUYEN, & H. KIM,& T. SENDOVA), *COMPETING DYNAMICS: ANALYZING MARKET SHARE IN A DUOPOLY*, Michigan State University

[2] `https://en.wikipedia.org/wiki/Normal_distribution`

[3] `https://www.thestandard.com.hk/breaking-news/section/4/222885/Three-bus-firms-apply-for-fare-increases,-ranging-from-6.5pc-to-9.5-pc`